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THE APPLICABILITY OF BAYES' THEOREM IN AUDIT RISK

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Abstract

The aim of this paper is to summarize the importance of the statistics in the financial audit area. Our main goal it is focused on "dissecting" the area of statistics to see what we can "steal" more from it for introducing it in the audit field. In the following paper we foresee a future direction of the audit for the next century. We try to prove that, if the auditor will used the Bayes' Theorem, then he will know for sure which is the possibility for a certain audit risk to appear, and which consequences can generate.

Keyword: financial audit; audit risk; Bayes' Theorem

JEL Classification: C11, M40, M42.

I. The connection between statistics and audit

Once with the advent of audit, also was taken into consideration the development of some procedures as simple for this activity, but in the same time to guarantee insurance and certification of these procedures. The involvement of the techniques and statistical methods in auditing was the result of a long process, assumed also by the professionals in our country. In both accounting and auditing, the role of the statistics is well defined regarding the principles in the field, see Barnett (2009), Tijms (2007), Steele (1992),

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Arens and Loebbecke (1981) and Smith (1976). Statistical methods to measure the audit risks that may arise in accounting and auditing continues to offer a certification increasingly more elaborate, due to this fact more and more accountants and auditors are starting to use the methods.

Our main goal it is focused on "dissecting" the area of statistics to see what we can "steal" more from her for introducing it in the audit field, for looking ahead to a future direction towards which he is moving.

In his book, Lawrence R. Dicksee Auditing: A Practical Manual for Auditors, published in 1907, states that the objective of an audit may be said to be threefold:

- the detection of fraud;
- the detection of technical errors;
- the detection of errors of principles.

As we know, the main objective of the independent auditor is to express an opinion on the fairness of financial position presented by the manager at a time and the financial results in a given period of time.

Although, apparently audit profession should be very simple, yet it's so complex and complicated. Due to this this fact the authors resort to different statistical methods to help them to determine more easily the risks that may arise during the formation of the opinion.

II. Bayes' Theorem

Why the Bayes' Theorem?

It is a really simple question, with an answer that it is, even, more easily to offer.

Bayes' Theorem is a supreme achievement for mankind or we can even name it a revelation, it all depends on how it is viewed the mathematics. No one should go through life without knowing what the Bayes' Theorem is and how it can be applied to every single element of our life. The first interesting thing is that the theorem is applicable in any field and in this way it can show us what to expect in any given situation. It should be emphasized, however, that science always investigating new concepts, new hypothesis so that science does not always provide 100% certification.

The Reverend Thomas Bayes first discovered the theorem that know bears his name. It was written up in a paper *An Essay Toward Solving a Problem in the Doctrine of*

Chances. The paper was found by his friend Richard Price, after the reverend died, and who had published it posthumously in 1763 (Bolstad W., 2007, pg. 6).

Bayes' Theorem is not the only one used in the audit field but as some authors proved to be the more efficient in the audit area we decided also to use it. In practice we find and the Stinger's bound that it is a non-parameter 100 (1-α) % and is used to find the upper limit of confidence for a fraction of errors in a population of crowds. It is easy to use but in many cases it is overly conservative. Moreover the 1989 *National Research Council's panel report on Statistical Models and Analysis in Auditing stated that* „...the formulation of the String bound has never been satisfactorily explained". This raport is an excellent survey of this and related problems. (Meeden G., Sargent D., 2005, pg. 3).

Another example of the efficiency of the Bayes' theorem was proved in 2007 by Kirk, Manalopoulos and Spathis who conducted a research entitled *Data Mining techniques for the detection of fraudulent financial statements* where they experimented with 3 statistical models: the neural network; the algorithm „decision trees" and the Bayesian network. The results obtained in the validation model, as a consequence of the applicability of the models for classify the firms into firms with fraudulent financial statements and firms with the financial statement correct that were reported based on the information from the final financial reporting, were the follows (Stroe I.A, Medinţu D.N., 2011, pg. 19):

- accuracy of the model and algorithm „decision trees" reached a level of 73.60%;
- accuracy of the model based on neural network reached 80%;
- accuracy of the model based on bayesian network has reached a level of 90.30%.

The accurate methods can't be 100% sure, but if we had to choose a model by which to go, we will always choose the one that gives us a higher rate of certainty, comparing the one that give us a lower percent of certainly.

The Bayesian model in the assessment of the audit risk express the possibility to use the probabilities with the personal estimated values and to modify the objective as a new data appear, because the elements of uncertainty are numerous and subjective, and may be revised as a result of the acquisition of new information (Genete D., 2006, pg. 26).

The power of Bayes' theorem lies in the fact that it is related to the quality of the interest, the probability that the hypothesis is true given the data, to the term that we have a better chance of being able to assign, the probability that we would have observed the

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measured data if the hypothesis is true. (Sivia D., Skilling J., 2006, pg. 6). Bayes' theorem is a first step in determining the probability that an event „A" in the situation when it is known that the appearance of it is affected by the achievement of another event „B" which is independent. In other words, the event that is associated with a possibility depends on the level of knowing (and knowledge) to the awarding possibility. When we know the phenomenon that we are analyzing it, it will increase based on the new information that it is bring to the phenomenon, thereby the possibility that was attributed to the event changes.

The formula of Bayes' Theorem is the following (Sasu L. *Modele grafice probabilistice – note de curs*, 2013):

$$P(E|C) = \frac{P(C|E)P(E)}{P(C)} \quad (1)$$

Equation 1 – The formula of Bayes'

Where:

- P(E|C) is called *posteriori* probability. With other words it expresses that the experience is the one that will determine what is true and what is false.
- P(E) is called *priori* probability. It is based on theoretical knowledge of the knowledge and is not based on their empirical observation is that the event „E" that is going to happen since event „C" has already happened;
- P(C|E) is called *likelihood*. Check the most probable value (the best one) of the event „C" it is the one for whom the event „E" has a maximum value;
- P(C) is called a *record* (proof). Is the event „C" that is known to already happened.

This theorem is a conditional probability for event „E" that will happen only if the event „C" has already happened. This means that whenever we add new information, if we know that the event „C" will happen, no matter what, what are the events „E" that result from adding new information.

Knowing all this we can express the equation in a more plastic (artistic) way:

$$P(Hypotesis|Evidence) = \frac{P(Evidence|Hypotesis)P(Hypotesis)}{P(Evidence)} \quad (2)$$

Equation 2 The artistic formula of Bayes

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We can conclude that Bayes' theorem represent the update way of confidence in the hypothesis „E" under the influence of new evidence „C".

III. How does it work?

In the following paragraphs we will try to illustrate how to apply Bayes' theory in practice and to demonstrate that, contrary to public opinion, it is easy to apply in practice. All that is needed is the will, determination and to provide to customers the results that they are waiting for it.

Exemplifying theorem will be made in two practical cases. The methodology used in this study paper is adapted to meet the objectives and the aims of the work. The main method used in the preparation of this case study was the collection of theoretical information from sources such as books, articles, journals and internet. For the examples of the audit risk, had open discussions with various chartered accountants and financial auditing professionals.

Example 1

An auditor on his way to audition company thinks that it has 3 options that will meet within the company, namely: a control risk low, a control risk medium or a control risk high. With other words, he may find that accounting systems and internal controls are operating effectively or that is not working effectively. Through the preliminary assessment of control risk will detect and prevent material misstatements that may occur in the company. Along the way, he will meet with another expert who (without knowing that the auditor will audit the entity X) tells him that the entities' X accounting systems and internal controls are not effective. If before meeting with experts, the auditor started from the premise that the control risk is high in the entity X, after meeting with the expert, he it revises its opinion and conclude that control risk is low. At this time a conflict happens to the auditor because he knows that he should not be influenced by others, but knowing that the person he meet is an expert has no way to disregard the new information.

In the following, we will prove using Bayes' theorem what will happen if the auditor considers the new information available from expert or what will happen if he will not take this information into account. It should be noted that this example is a model demonstrate the applicability of Bayes' theorem in audit risk and any auditor before they

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form an opinion should analyze all data within the company and they shouldn't form an opinion on what another person tells them..

We denote by R_i the event on the opinion on risk control, ie $1 \leq i \leq 3$ – this represents that the risk control can be any of the three variants (low, medium and high), but given that the auditor starts from assume that control risk is high it is denoted by i. Initially $P(R_i)=\frac{1}{3}$. We denote with P_{ij} the event that the auditor meet with the expert which provides him with new information (j) (involuntary) about the entity X, this is the element that modifies the opinion about the control rick. From all this results that:

$$P(P_{ij}|R_k) = \begin{cases} 0, & if\ i = j \\ 0, & if\ j = k, i = j \\ \frac{1}{2}, & if\ i = j, j \neq k \\ 1, & if\ i \neq k, j \neq k, i \neq j \end{cases} \quad (3)$$

Without restricting the generality of events that happens we believe that the auditor assumes that control risk is high (i = 1), and expert opinion influences which then becomes that control risk is low (j = 3). For starters, the auditor does not want to be influenced by expert opinion, but since he acknowledges was made known to (accidentally) this new information, he can't ignore them:

$$P(R_1|P_{13}) = \frac{P(P_{13}|R_1)P(R_1)}{P(P_{13})} = \frac{1}{6} * 2 = \frac{1}{3} \quad (4)$$

$$\text{Where } P(P_{13}|R_1)P(R_1) = \frac{1}{2} * \frac{1}{3} = \frac{1}{6}; \text{ also}$$

$$P(P_{13}) \sum_{i=3}^{3} P(P_{13}|R_i)P(R_i) = \frac{1}{2}, \text{therefore } P(R_1|P_{13}) = \frac{1}{3}. \quad (5)$$

This proves the fact that no matter how much he would like the opinion of the auditor on the fact that that the controls risk is high, this it changes when the new information was brought to their attention, by the experts in the area.

So, we have $P(R_3|P_{1\ 3})=0$ and through complementarity we obtain $P(R_2|P_{1\ 3})=1-P(R_1|P_{1\ 3})-P(R_3|P_{1\ 3})=1-\frac{1}{3}-0=\frac{2}{3}=2*P(R_1|P_{1\ 3})$ from here we can observe that result that they are two more chances that the information which the audit got it from the expert to be true, and if the auditor will changes his initial opinion, that the control risk is high, he has twice more chances to testify the fact that control risk is medium or low. Of course,

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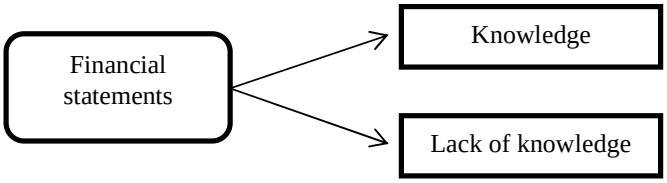
this does not means that, information from experts is not false and control risk within the entity X to be high, but in the absence of other knowledge is better to change their original opinion.

Example 2

The first time when the financial auditor enter in a company which he is going to audit it, will think that he has two choices: to find that the annual financial statements are properly prepared or that the financial statements aren't properly prepared. In the meantime, he finds out that the company management has changed. From this situation may arise two possibilities: that the new leadership has knowledge in the area that he was chosen, or he has a total lack of knowledge in the area. The experience, the knowledge management, as well as the changes that arise in the management in the audit period, e.g. the inexperience of the new management may affect the financial statements - which have been a general inherent risk. This statement can be seen in the figure below.

Figure 1 Initial situation



Suppose that after the financial auditor takes knowledge of this fact he is thinking of two possibilities that may overcome from the given situation of the company, from the fact that the manager has knowledge of financial statements and from the fact that the manager has a lack of knowledge in the given are:

- Financial statements correct – that will continue to be prepared correctly or will start to be prepared wrong given that the manager has knowledge in the field;
- Financial statements wrong – that will start to be prepared correctly or will continue to be prepared wrong given that the manager has a lack of knowledge in the field.

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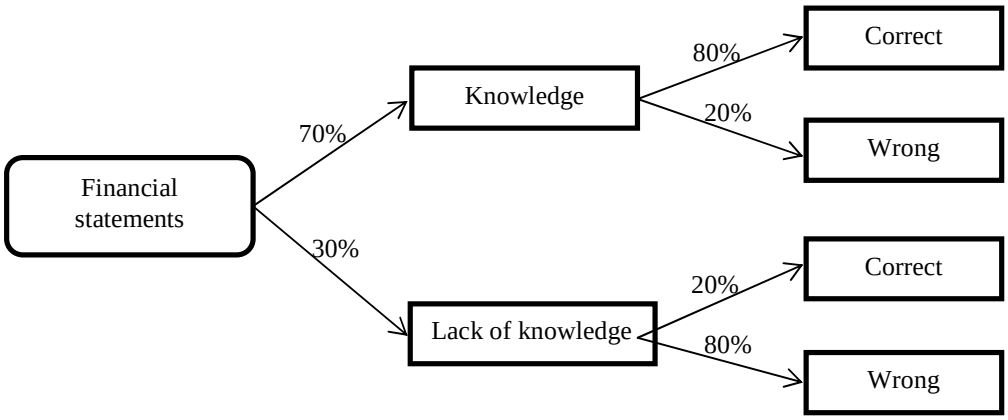
Do to this new situation we'll continue the next figure in the following way:

Figure 2 Model of application of Bayes' Theorem in audit



Observation: It states that the percentages obtained are based on professional competence and forecasts based on several talks with chartered accountants and financial auditing professionals and this can vary from a professional to another.

Explication of the second figure:

- probability of 70% and 30% - i.e. once the auditor meets the financial statements until the moment when he will form an opinion on the level of trust that will be given on them, taking into account the finding news that the company's management has changed is called the unconditional probability. I.e., the independent auditor will form an opinion at the end of favorable or unfavorable, whether new management has experience or not, this means that the event „knowledge / lack knowledge" already happened regardless of future ones. Values and scoring this indicator is thus P (E) = 70% when the new leadership experience helps or not the financial statements, i.e. P (E) = 30% for the case when the inexperience of new leadership helps or not the financial statements.
- probability of 80% and 20% - that is, since the leadership change and thereby affecting the financial statements in the positive or in the negative way, this period is expressing the conditional probability, i.e. the change in the company wouldn't happen probably that the change produce would not be greatly affect the final result, but since

they introduced the new information to our main event, they generate another hypotheses. In this way, seems that the financial statements can be positively or negatively affected by the change in the management. There is the possibility that the lack of knowledge is likely to affect the final preparation of the financial statements. The values and the scoring of this indicator is as follows:

- P(C)=80% - when the financial statements are properly drawn up in the end, and the knowledge of the new leadership helps in elaboration them;
- P(C)=20% - when the financial statements are wrong drawn up in the end, and the knowledge of the new leadership doesn't helps in elaboration them;
- P(C)=20% - when the financial statements are properly drawn up in the end, and the lack of knowledge of the new leadership helps in elaboration them;
- P(C)=80% - when the financial statements are wrong drawn up in the end, and the lack of knowledge of the new leadership doesn't helps in elaboration them

- joint probability - the likelihood that the financial statements are prepared correctly is 80% given (i.e. due to the fact) that the management of the company has changed. The values and the scoring of this indicator is thus P (C | E) = P (C) * P (E) = (70%) (80%) = 56%. This means that if at the end of the audit period there is a verisimilarly of 56% as that the financial statements are properly drawn up regarding the fact that the new management have knowledge in the area. As demonstrated, the credibility of the financial statements to be prepared correctly is a moderate degree of the confidence even if the new management has knowledge in the area. On the other hand, if we calculate which is the credibility that the financial statements are prepared correctly even if the manager's lack of knowledge is P (C | E) = P (C) * P (E) = (30%) (20%) = 6%. A very low confidence level is given in this situation, must be borne in mind that if we gather the four probabilities P (C | E) must be the total of 100%.

Given the above data, Bayes' theorem enables us to answer the following question: If at the end of the year, the financial statements of the company are properly drawn up, what is the probability (possibility) that the new leadership skills to affect them?

Knowing that the unconditionally event that the financial statements are properly drawn up at the end of the year, are influence by the new leadership with 70%, but early, at the beginning of the year, evidence that the year before the financial statements have been prepared correctly, then we change the perspective that the experience of the new

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management would influence the proper preparation of the financial statement, to see this we can apply Bayes' theorem.

$$P(E|C) = \frac{P(C|E)P(E)}{P(C)} = \frac{(80\%) * (70\%)}{(80\%) * (70\%) + (30\%)(20\%)} = \frac{56\%}{56\% + 6\%} = \frac{56\%}{62\%} = 90.32\% \quad (6)$$

We note that the conditional probability increased, knowing is noted that the financial statements at the end of the year, with 90.32%.

So we went to the unconditional probability where we had no information to condition the new leadership and experience which is 70%, when we observed that the financial statements have been prepared correctly (since they are very much dependent the knowledge of the person who prepares them) then we can say with certainty that the new leadership skills to prepare accurate financial statements at the end of the year grow to 90.32% from 70%, so it is necessary to have experience in the field to prepare financial statements accordance with the international standards and with the law.

We can conclude that there is 90.32% chance that the risk inherent to intervene if the new leadership does not have knowledge in the field, this means that it is inherent in the entity level.

IV. A final note

Given the two examples shown above we can conclude the two following statements:

- In the first example, since we do not have further information occurred, the auditor will remain of the opinion that control of the entity's risk is low to medium;
- In the second example demonstrated that knowledge in the field of new leadership will influence inherent risk concluding that it is high.

Considering the following table we can say that there are 2.25 times more likely to:

- undetected risk if the control risk is low and the inherent risk is high to medium
- undetected risk if the control risk is medium the inherent risk is high to low.

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Table 13 Appraisement of the undetected risk

		Control risk		
		High	**Medium**	**Low**
Inherent risk	**High**	Really Low	Low	Medium
	Medium	Low	Medium	High
	Low	Medium	High	Really high

Source: Cosserat G.W, *Modern Audit,2nd edition*, John Wiley & Sons, England, 2005, pp. 138

If we are taking into consideration the probabilities theorem we can appreciate that there is a 56.26% chance that the two situations that we concluded above, may happen. As we can notice the probability is quite high, in this moment the auditor has to gadder more evidence to see in which part to tip the balance.

This paper offer to the readers a short guidance of how we can use the Bayes' theorem in the audit rick, it is a tool to fulfill the need of the professional that it is curious regarding the applicability of the theorem in the audit risk. Auditors have a great responsibility to respond to all the audit risks. In the paper we proved what is the probability for an event to happen, in a given situation, when we are adding more information to the main hypothesis. Using the Bayes' theorem, the time that the auditor is using with preparing all the procedures will become shorter, a fact that everybody wants to achieve. There are a lot of methods that we can use to detect and prevent the audit risk but, as we proved above, the Bayes' theorem is the best one to use. We try to fulfill the main goal of this paper and in this way the information from the paper it can be available and accessible to a wide range of experts who might want to short their time from working time.

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